

St Nicholas' Primary



Second Level

Numeracy and Mathematics Booklet



A Guide for Parents and Pupils

Introduction

What is Numeracy?

Numeracy is a skill for life, learning and work. Having well-developed numeracy skills allows young people to be more confident in social settings and enhances enjoyment in a large number of leisure activities.

Curriculum for Excellence

The better your child knows the basics, the easier it will be for him/her to make progress. It is important that your child practises these basic facts at home - namely quick recall of number bonds, place value, times tables, measurement, time and money and is encouraged to use them in everyday life.

What is the purpose of the booklet?

This booklet has been produced in collaboration with cluster schools to give guidance to parents/carers on how certain common topics are taught within the Mathematics curriculum following the Curriculum for Excellence guidelines.

The mathematics experiences and outcomes are structured within three main organisers, each of which contains a number of subdivisions:

Number, Money and Measure

- Estimation and rounding
- Number and number processes
- Multiples, factors and primes
- Powers and roots
- Fractions, decimal fractions and percentages
- Money
- Time
- Measurement
- Mathematics - its impact on the world, past, present and future
- Patterns and relationships
- Expressions and equations

Shape, position and movement

- Properties of 2D shapes and 3D objects
- Angle, symmetry and transformation

Information Handling

- Data and analysis
- Ideas of chance and uncertainty

From the early stages, children should experience success in mathematics and develop the confidence to take risks, ask questions and explore alternative solutions without fear of being wrong. Children will be exploring and applying mathematical concepts to understand and solve problems, explaining their thinking and presenting their solutions to others in a variety of ways. At all stages, an emphasis on collaborative learning will encourage children to reason logically and creatively through discussion. Children will show evidence of progress through their skills in collaborating and working independently as they explore and investigate mathematical problems.

As children develop concepts within mathematics there will be continual reinforcement and revisiting in order to maintain progression.

How can this booklet be used?

If you are helping your child with homework, you can refer to the booklet to see what methods are being taught.

Why do some topics include more than one method?

In some cases the method used will be dependent on the level of difficulty of the question.

For mental calculations, children should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.

There are many opportunities to develop mathematical concepts through other areas of the curriculum or contexts out with school.

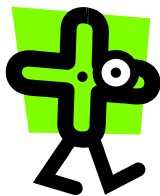
Adapted from 'Numeracy Booklet - A Guide for Parents'
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Table of Contents

Topic	Page Number
Addition	4
Subtraction	5
Multiplication	6- 8
Division	9
Integers	10
Order of Calculations (BODMAS)	11
Equations	12
Estimation	13, 14
Time	15, 16
Fractions	17, 18
Percentages	19-21
Ratio	22-24
Proportion	25
Information Handling	26-31
Length	32
Weight	33
Volume	34
Area	35
Mathematical literacy (key words)	36-38
Useful Tools	39

Addition

Mental strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example Calculate $64 + 27$

Method 1 Add tens, then add units, then add together

$$60 + 20 = 80 \qquad 4 + 7 = 11 \qquad 80 + 11 = 91$$

Method 2 Split up number to be added (last number 27) into tens and units and add separately.

$$64 + 20 = 84 \qquad 84 + 7 = 91$$

Method 3 Round up to nearest 10, then subtract

$$64 + 30 = 94 \quad \text{but } 30 \text{ is } 3 \text{ too much so subtract } 3;$$

$$94 - 3 = 91$$

Written Method

When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down units, carry tens under the line.

Example Add 3032 and 589

$\begin{array}{r} 3032 \\ +589 \\ \hline \end{array}$	\rightarrow	$\begin{array}{r} 3032 \\ +589 \\ \hline \end{array}$	\rightarrow	$\begin{array}{r} 3032 \\ +589 \\ \hline \end{array}$
$\begin{array}{r} 1 \\ \hline \end{array}$	\rightarrow	$\begin{array}{r} 21 \\ \hline \end{array}$	\rightarrow	$\begin{array}{r} 621 \\ \hline \end{array}$
$\begin{array}{r} 3621 \\ \hline \end{array}$				
$2 + 9 = 11$	$3 + 8 + 1 = 12$	$0 + 5 + 1 = 6$	$3 + 0 = 3$	

Subtraction



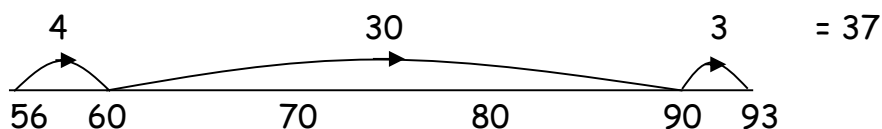
We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Mental Strategies

Example Calculate $93 - 56$

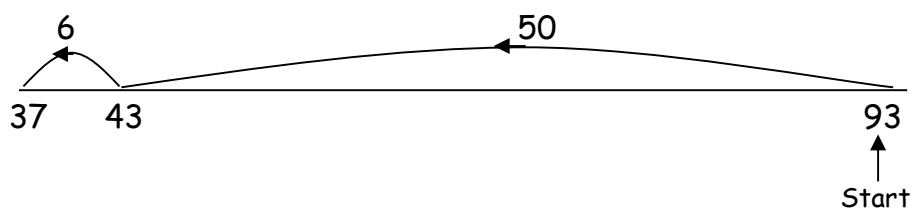
Method 1 Count on

Count on from 56 until you reach 93. This can be done in several ways e.g.



Method 2 Break up the number being subtracted

e.g. subtract 50, then subtract 6 $93 - 50 = 43$
 $43 - 6 = 37$



Written Method

Example 1 $4590 - 386$

$$\begin{array}{r} 4590 \\ - 386 \\ \hline 4204 \end{array}$$

Example 2 Subtract 692 from 3000

$$\begin{array}{r} 2991 \\ \cancel{3000} \\ - 692 \\ \hline 2308 \end{array}$$

We do not
"borrow and
pay back".

Important steps for example 1

1. Say "zero subtract 6, we cannot do"
2. Look to next column exchange one ten for ten units.
3. Then say "ten take away six equals four"
4. Normal subtraction rules can be used to then complete the question.

Multiplication 1



It is essential that you know all of the multiplication tables from 1 to 10. These are shown in the tables square below.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Mental Strategies

Example Find 18×6

Method 1

$$10 \times 6 = 60$$

$$8 \times 6 = 48$$

$$60 + 48 = 108$$

Method 2

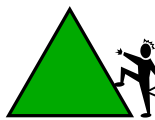
$$20 \times 6 = 120$$

20 is 2 too many
so take away 6×2

$$120 - 12 = 108$$

Multiplication 2

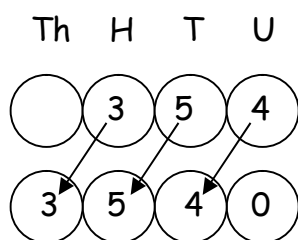
Multiplying by multiples of 10 and 100



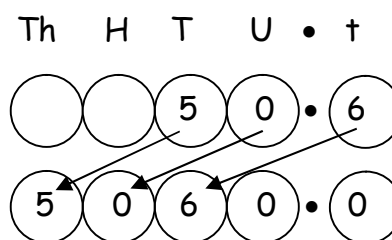
To multiply by **10** you move every digit **one** place to the left.

To multiply by **100** you move every digit **two** places to the left.

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100



$$354 \times 10 = 3540$$



$$50.6 \times 100 = 5060$$

(c) 15×30

To multiply by 30,
multiply by 3,
then by 10.

$$15 \times 3 = 45$$

$$45 \times 10 = 450$$

$$\text{so } 15 \times 30 = 450$$

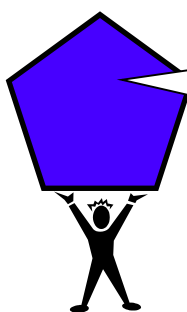
(d) 56×200

To multiply by
200, multiply by 2,
then by 100.

$$56 \times 2 = 112$$

$$112 \times 100 = 11200$$

$$\text{so } 56 \times 200 = 11\,200$$



We may also use these rules for multiplying decimal numbers. Decimal points do not move!

Example 2 (a) 2.3×20 (b) 1.12×40

$$2.3 \times 2 = 4.6$$

$$4.6 \times 10 = 46.0$$

$$\text{so } 2.36 \times 20 = 47.2$$

$$1.12 \times 4 = 4.48$$

$$4.48 \times 10 = 44.8$$

$$\text{so } 1.12 \times 40 = 44.8$$

Multiplication 3

Multiplying by written methods

Example 1 Multiply 354 by 19

$$\begin{array}{r}
 354 \\
 \times 19 \\
 \hline
 3186 \leftarrow 354 \times 9 \\
 +3540 \leftarrow 354 \times 10 \\
 \hline
 6726 \\
 \hline
 1
 \end{array}$$

The 'zero' is placed in the units column so that we can hold the tens place, then multiply as normal by the 'ten', in this case '1'

Example 2 Multiply 456 by 32

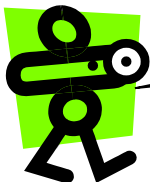
$$\begin{array}{r}
 456 \\
 \times 32 \\
 \hline
 912 \leftarrow 456 \times 2 \\
 +13680 \leftarrow 456 \times 30 \\
 \hline
 14592 \\
 \hline
 1
 \end{array}$$

The 'zero' is placed in the units column so that we can hold the tens place, then multiply as normal by the 'ten', in this case '3'

*Please note that carrying would be expected in the written calculation, but has been omitted for clarity.

** To multiply by a three digit number you simply add two zeros to hold the 'hundreds' place on the third line of the calculation and multiply by the 'hundred'

Division



You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.

Written Method

18 divided by 3 can be shown as...

$$18 \div 3 = \quad \text{or} \quad \begin{array}{r} 3 \overline{)18} \\ \end{array} \quad \text{or} \quad \frac{18}{3} \quad \text{or} \quad \frac{1}{3} \text{ of } 18$$

Example 1 There are 56 pupils in P7, shared equally between 2 classes. How many pupils are in each class?

$$\begin{array}{r} 28 \\ 2 \overline{)56} \\ \end{array}$$

There are 28 pupils in each class

Example 2 Divide 474 by 3

$$\begin{array}{r} 158 \\ 3 \overline{)474} \\ \end{array}$$

Always carry the remainder to the next column.

Example 3 A jug contains 2.64 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

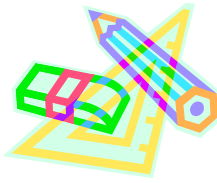
$$\begin{array}{r} 0.33 \\ 8 \overline{)2.64} \\ \end{array}$$

Each glass contains
0.33 litres

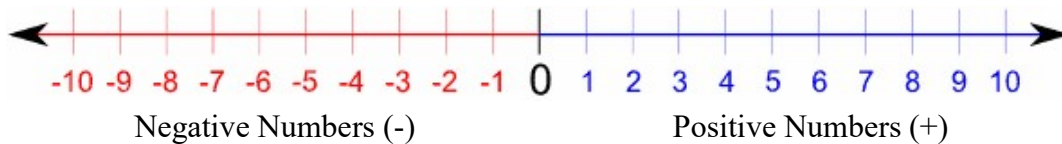
The decimal points must stay in line.

If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Integers - Adding and Subtracting



An integer is what is more commonly known as a whole number. It may be positive, negative, or the number zero, but it must be whole.



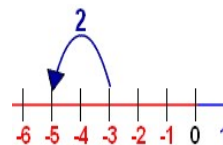
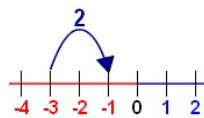
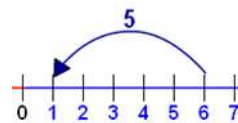
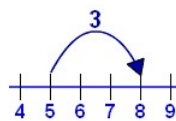
Remember: No Sign in front of a number means it is positive

Adding and Subtracting positive numbers

A number line may be used if pupils are finding questions difficult to do mentally

Examples $5+3 = 8$

$6-5 = 1$



If you add a positive number you move to the right on a number line.
If you subtract a positive number you move to the left on a number line.
Always start from the position of the first number.

Adding or subtracting negative numbers.

Adding a negative number is the same as subtracting:

Example $7 + (-3)$ is the same as $7 - 3 = 4$

General rule $a+(-b) = a - b$

Subtracting a negative number is the same as adding:

Example $(-5) - (-2)$ is the same as $(-5) + 2 = -3$

General rule $a-(-b) = a + b$

Order of Calculation (BODMAS)

Consider this: What is the answer to $2 + 4 \times 5$?

Is it	$(2+4) \times 5$	or	$2 + (4 \times 5)$
	$= 6 \times 5$		$= 2 + 20$
	$= 30$		$= 22$

The correct answer is 22.



The **BODMAS** rule tells us which operations should be done first.

BODMAS represents:

(B)rackets

(O)rder

(D)ivide

(M)ultiply

(A)dd

(S)ubtract

Therefore in the example above multiplication should be done before addition. (Note order means a number raised to a power such as 2^2 or $(-3)^3$)

Scientific calculators are programmed with these rules, however some basic calculators may not, so take care.

Example 1 $15 - 12 \div 6$ BODMAS tells us to divide first

$$= 15 - 2$$

$$= 13$$

Example 2 $(9 + 5) \times 6$ BODMAS tells us to work out the

$$= 14 \times 6$$

$$= 84$$

brackets first

Example 3 $18 + 6 \div (5-2)$ Brackets first

$$= 18 + 6 \div 3$$

$$= 18 + 2$$

$$= 20$$

Then divide
Now add

Equations

$$a^2 + b^2 = c^2$$

An equation is a statement or mathematical expression which says one side is equal to the other side.

Think of each side of the equation as one side of a set of scales which says one side is equal to the other.

This method is called Balancing.

RULES

Letters to the left, numbers to the right. If you change side you change sign

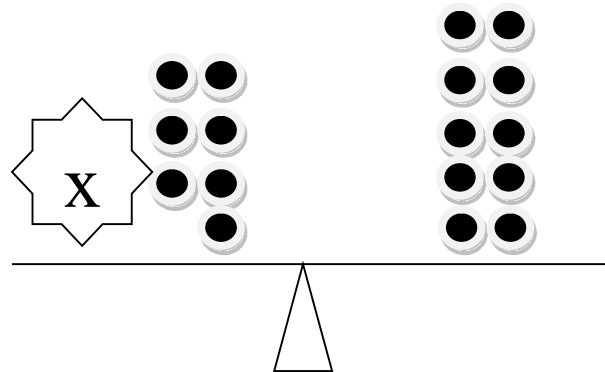
Example 1

Solve for x

$$x + 7 = 10$$

$$x = 10 - 7$$

$$x = 3$$



Example 2

$$4x = 48$$

$$x = 48 \div 4$$

$$x = 12$$

Example 3

$$2x + 3 = 9$$

$$2x = 9 - 3$$

$$2x = 6$$

$$x = 6 \div 2$$

$$x = 3$$

identify the number +3 must change sides and sign

+3 changes to -3

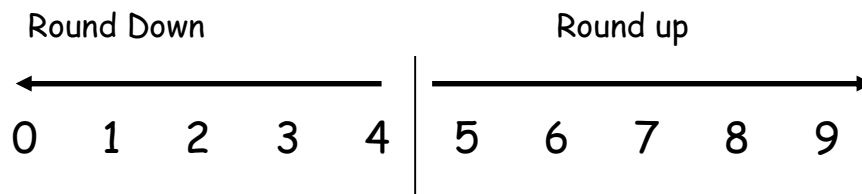
Important points to remember

The letter x should be written differently from a multiplication sign, but remember other letters may also be used. Only one equals sign per line. Equals signs should be kept beneath each other in line.

Estimation : Rounding Whole Numbers



Numbers can be rounded to give an approximation.
IMPORTANT RULE
 We always round up for 5 or above
 786 rounded to the nearest 10 is 790.



We can round as follows -

- Round 2 digit whole numbers to the nearest 10
- Round 3 digit whole numbers to the nearest 10 or 100
- Round 4 digit whole numbers to the nearest 10, 100 or 1000

Example

652 rounded to the nearest 10 is 650

785 rounded to the nearest 10 is 790

2652 rounded to the nearest 100 is 2700

7845 rounded to the nearest 100 is 7800

2652 rounded to the nearest 1000 is 3000

7845 rounded to the nearest 1000 is 8000

The same principle applies to rounding decimal numbers.

3.64 to the nearest tenth is 3.60 or 3.6

In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right (the "check digit") - if it is 5 or more round up.

Estimation : Calculation



We can use rounded numbers to give us an approximate answer to a calculation. This allows us to check that our answer is sensible.

Example 1

Tickets for a P7 concert were sold over 4 days. The number of tickets sold each day was recorded in the table below. How many tickets were sold in total?

Monday	Tuesday	Wednesday	Thursday
48	23	18	36

Estimate = $50+20+20+40=130$ therefore the exact answer should be about 130.

Calculate:

$$\begin{array}{r}
 48 \\
 23 \\
 18 \\
 +36 \\
 \hline
 125
 \end{array}$$

Answer = 125 tickets

Example 2

A bar of chocolate weighs 42g. There are 20 bars of chocolate in a box. What is the total weight of chocolate in the box?

Estimate = $40 \times 20 = 800\text{g}$

Calculate:

$$\begin{array}{r}
 42 \\
 \times 20 \\
 \hline
 0 \\
 840 \\
 \hline
 840
 \end{array}$$

Answer = 840g

Time 1



Time may be expressed in 12 or 24 hour notation.

Time Facts - What you should already know!

60 seconds	=	1 minute
60 minutes	=	1 hour
24 hours	=	1 day
7 days	=	1 week
52 weeks	=	1 year
365 days	=	1 year
366 days	=	1 leap year

How many days are in each month? Learn this rhyme, it works!

Thirty days has September,
 April June and November,
 All the rest have 31 days clear,
 Except February alone which has
 28 days clear and
 29 in a leap year.

12-hour clock Time can be displayed on a clock face, or digital clock.



05:15

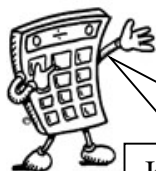
These clocks both show fifteen minutes past five, or quarter past five.

When writing times in 12 hour clock, we need to add am or pm after the time.

am is used for times between midnight and 12 noon (morning)

pm is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock



In 24 hour format the hours are written as numbers between 00 and 24. Midnight is expressed as 00 00 or 24 00.
 After 12 noon the hours are noted as 13, 14, 15...etc.

Hours → ← Minutes

	hh	mm
Midnight	00	00
1.00am	01	00
5.00am	05	00
9.00am	09	00
10.00am	10	00
12 noon	12	00
1.00pm	13	00
4.00pm	16	00
7.00pm	19	00
9.15pm	21	15
10.30pm	22	30
11.45pm	23	45

Time 2



We can work out durations of time by "counting on". This is a simple method to learn and is useful for timetables or schedules

Time Calculations

Example 1 How long is it from 9.30am to 11.15 am

Method - Working

9.30 → 10.00 → 11.00 → 11.15
 (30mins) + (1hr) + (15mins) = 1hr 45 minutes

****TIME SHOULD NOT BE CALCULATED USING SUBTRACTION****

Example 2 How long is it from 13 55 to 16 30

13 55 → 14 00 → 16 00 → 16 30
 (5mins) + (2 hrs) + (30mins) = 2hrs 35 minutes

	1st	2nd	3rd	4th	5th	6th
Depot	07:30	07:45	08:00	08:15	08:30	08:45
Green St	07:40	07:55	08:10	08:25	08:40	08:55
High St	07:45	08:00	08:15	08:30	08:45	?
Central Park	07:48	08:03	08:18	08:33	08:48	09:03

Reading timetables

When reading timetables you often have to convert to and from 24 hours clock.

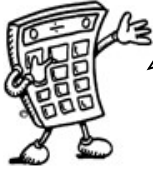
To convert from 24 hour time to 12 hour time:

- If the hour is 13 or more, subtract 12 from the hours and call it pm Otherwise it is am
- If the hour is 12, leave it unchanged, but call it pm
- If the hour is 00, make it 12 and call it am
- Otherwise, leave the hour unchanged and call it am

To convert from 12-hour time to 24-hour time:

- If the pm hour is from 1 through 11, add 12.
 - If the pm hour is 12, leave it as is.
 - If the am hour is 11, or 10, leave it as is
 - If the am hour is a single digit, place a 0 before it (1.00am =01 00)
 - Otherwise, leave the hour unchanged.
- Then drop the am or pm of course.

Fractions 1

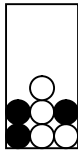


Addition, subtraction, multiplication and division of fractions are studied in mathematics. However, the examples below may be helpful in all subjects.

Understanding Fractions

Example

A jar contains black and white sweets.



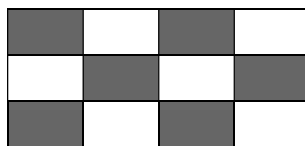
What fraction of the sweets are black?

There are 3 black sweets out of a total of 7, so $\frac{3}{7}$ of the sweets are black.

Equivalent Fractions

Example

What fraction of the flag is shaded?



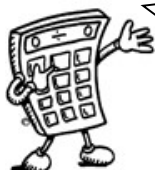
6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions 2

Simplifying Fractions



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.

To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same number.

Example 1

$$(a) \quad \frac{20}{25} \xrightarrow{\div 5} \frac{4}{5}$$

$$(b) \quad \frac{16}{24} \xrightarrow{\div 8} \frac{2}{3}$$

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in its **simplest form**.

Example 2 Simplify $\frac{72}{84}$ $\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (simplest form)

Calculating Fractions of a Quantity



To find the fraction of a quantity, divide by the denominator.

To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

Example 1 Find $\frac{1}{5}$ of £80

$$\frac{1}{5} \text{ of } £80 = £80 \div 5 = £16$$

Example 2 Find $\frac{3}{4}$ of 48

$$\frac{1}{4} \text{ of } 48 = 48 \div 4 = 12$$

$$\text{so } \frac{3}{4} \text{ of } 48 = 3 \times 12 = 36$$

To find $\frac{3}{4}$ of a quantity, start by finding $\frac{1}{4}$ then multiply by 3 (the numerator)

Percentages 1



Percent means out of 100.

A percentage can be converted to an equivalent fraction or decimal.

10% means $\frac{10}{100}$ simplified to $\frac{1}{10}$

10% is therefore equivalent to $\frac{1}{10}$ and 0.1

Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals.

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	0.01
10%	$\frac{1}{10}$	0.1
20%	$\frac{1}{5}$	0.2
25%	$\frac{1}{4}$	0.25
$33\frac{1}{3}\%$	$\frac{1}{3}$	0.333...
50%	$\frac{1}{2}$	0.5
$66\frac{2}{3}\%$	$\frac{2}{3}$	0.666...
75%	$\frac{3}{4}$	0.75
100%	1 whole	1.0

Percentages 2



There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

Non- Calculator Methods

Method 1 Using Equivalent Fractions

Example Find 25% of £160

$$25\% \text{ of } \pounds 160 = \frac{1}{4} \text{ of } \pounds 160 = \pounds 160 \div 4 = \pounds 40$$

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

$$1\% \text{ of } 200\text{g} = \frac{1}{100} \text{ of } 200\text{g} = 200\text{g} \div 100 = 2\text{g}$$

$$\text{so } 9\% \text{ of } 200\text{g} = 9 \times 2\text{g} = 18\text{g}$$

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example Find 70% of £35

$$10\% \text{ of } \pounds 35 = \frac{1}{10} \text{ of } \pounds 35 = \pounds 35 \div 10 = \pounds 3.50$$

$$\text{so } 70\% \text{ of } \pounds 35 = 7 \times \pounds 3.50 = \pounds 24.50$$

Percentages 3

Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1 Find 23% of £15000

$$23\% = 0.23 \text{ so } 23\% \text{ of } \pounds 15000 = 0.23 \times \pounds 15000 = \pounds 3450$$



This method does not use the % button on calculators. The methods usually taught in mathematics departments are all based on converting percentages to decimals.

Example 2 House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

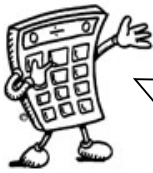
$$19\% = 0.19 \quad \text{so} \quad \text{Increase} = 0.19 \times \pounds 236000 \\ = \pounds 44840$$

$$\text{Value at end of year} = \text{original value} + \text{increase} \\ = \pounds 236000 + \pounds 44840 \\ = \pounds 280840$$

The new value of the house is £280840



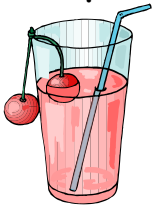
Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.

The ratio of water to cordial is 4:1

(said "4 to 1")

The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2



In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

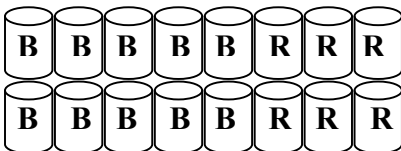
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



$$\begin{aligned} \text{Blue : Red} &= 10 : 6 \\ &= 5 : 3 \end{aligned}$$

To simplify a ratio, divide each figure in the ratio by a common factor.

Ratio 2

Simplifying Ratios (continued)

Example 2

Simplify each ratio:

(a) 4:6

(b) 24:36

(c) 6:3:12

(a) 4:6
= 2:3

Divide each
figure by 2

(b) 24:36
= 2:3

Divide each
figure by 12

(c) 6:3:12
= 2:1:4

Divide each
figure by 3

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg cement. Write the ratio of sand : cement in its simplest form

$$\begin{aligned} \text{Sand : Cement} &= 20 : 4 \\ &= 5 : 1 \end{aligned}$$

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

Fruit	Nuts
3	2
15	10

$\left. \begin{array}{c} 3 \\ 15 \end{array} \right\} \times 5$

 $\left. \begin{array}{c} 2 \\ 10 \end{array} \right\} \times 5$

So the chocolate bar will contain 10g of nuts.



Ratio 3

Sharing in a given ratio



Example

Lauren and Connor earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2. How much money did each receive?

Step 1 Add up the numbers to find the total number of parts

$$3 + 2 = 5$$

Step 2 Divide the total by this number to find the value of each part

$$90 \div 5 = \text{£}18$$

Step 3 Multiply each figure by the value of each part

$$3 \times \text{£}18 = \text{£}54$$

$$2 \times \text{£}18 = \text{£}36$$

Step 4 Check that the total is correct

$$\text{£}54 + \text{£}36 = \text{£}90 \quad \checkmark$$

Lauren received £54 and Connor received £36

Proportion



Two quantities are said to be in direct proportion if when one doubles the other doubles.
We can use proportion to solve problems.

It is often useful to make a table when solving problems involving proportion.

Example 1

A car factory produces 1500 cars in 30 days. How many cars would they produce in 90 days?

Days	Cars
30	1500
90	4500

$\left. \begin{array}{l} 30 \\ 90 \end{array} \right\} \times 3$ $\left. \begin{array}{l} 1500 \\ 4500 \end{array} \right\} \times 3$

The factory would produce 4500 cars in 90 days.

Example 2

5 adult tickets for the cinema cost £27.50. How much would 8 tickets cost?

Tickets	Cost	Working:
5	£27.50	$\begin{array}{r} \text{£}5.50 \quad \text{£}5.50 \\ 5 \overline{) \text{£}27.50} \quad \times 8 \\ \hline \text{£}44.00 \\ \hline 4 \end{array}$
1	£5.50	
8	£44.00	

The cost of 8 tickets is £44

Information Handling : Tables



It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	24	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. Often data is grouped in intervals.

Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

Mark	Tally	Frequency
16 - 20		2
21 - 25		7
26 - 30		9
31 - 35		5
36 - 40		3
41 - 45		2
46 - 50		2

Each mark is recorded in the table by a tally mark.

Tally marks are grouped in 5's to make them easier to read and count.

Information Handling : Bar Graphs/Histograms

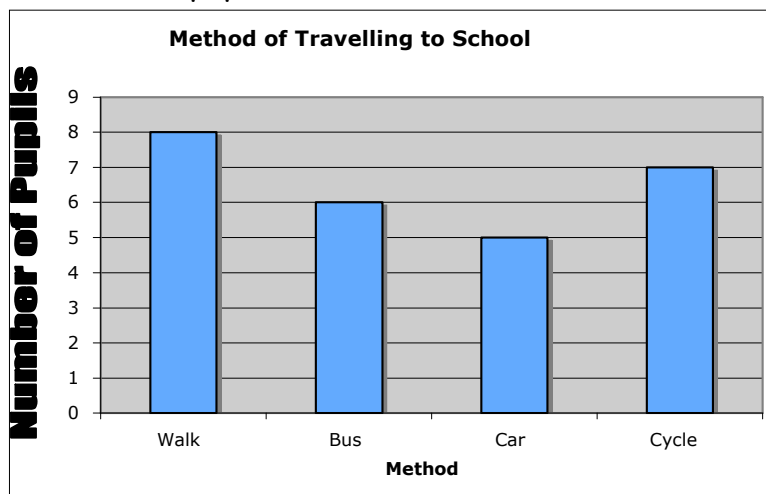


Bar graphs and Histograms are often used to display data. They must not be confused as being the same. Bar graphs are used to present discrete* or non numerical data* whereas histograms are used to present continuous data*. See key words for explanation of these terms

All graphs should have a title, and each axis must be labelled.

Example 1 Example of a Bar Graph

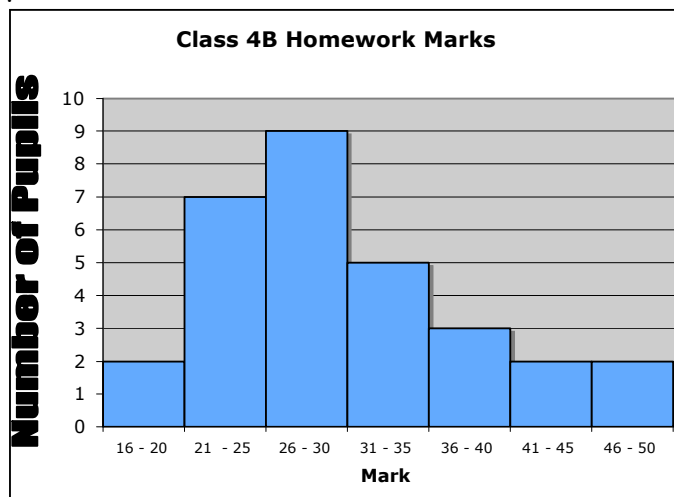
How do pupils travel to school?



An even space should be between each bar and each bar should be of an equal width. (also leave a space between vertical axis and the first bar.)

Example 2 Example of a histogram

The graph below shows the homework marks for Class 4B.



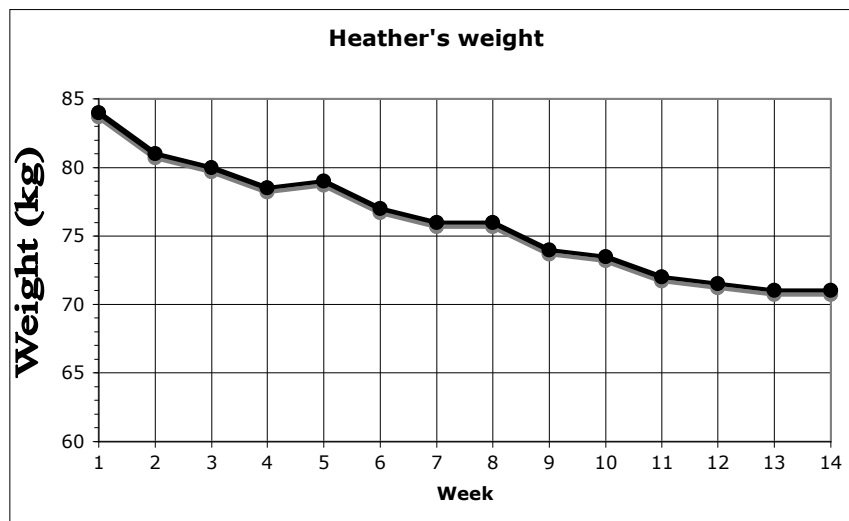
Important - there should be no space between each bar

Information Handling : Line Graphs



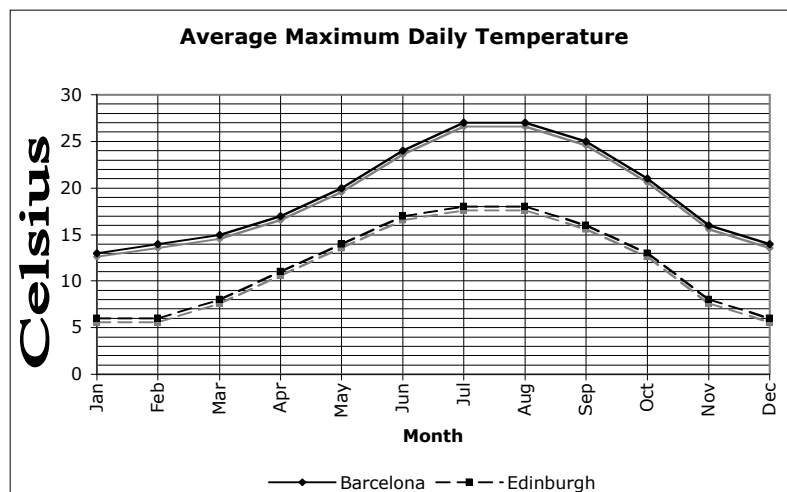
Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled. The trend of a graph is a general description of it.

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.



The trend of the graph is that her weight is decreasing.

Example 2 Graph of temperatures in Edinburgh and Barcelona.



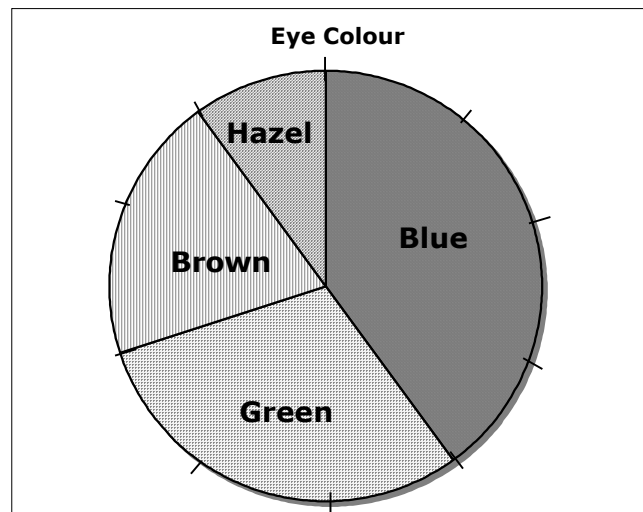
Information Handling : Pie Charts



A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example

30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.



How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the brown sector is 72° .
 so the number of pupils with brown eyes
 $= \frac{72}{360} \times 30 = 6$ pupils.

If finding all of the values, you can check your answers - the total should be 30 pupils.

Information Handling : Pie Charts 2

Drawing Pie Charts



On a pie chart, the size of the angle for each sector is calculated as a fraction of 360° .

Example: In a survey about school, a group of pupils were asked what was their favourite subject. Their answers are given in the table below. Draw a pie chart to illustrate the information. This would be done using a protractor.

Subject	Number of people
Mathematics	28
Home Economics	24
Music	10
Physics	12
PE	6

Total number of people = 80

$$\text{Mathematics} = \frac{28}{80} \rightarrow \frac{28}{80} \times 360^\circ = 126^\circ$$

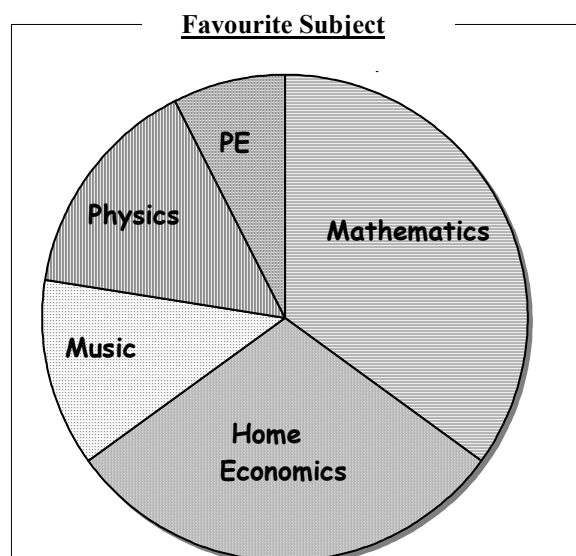
$$\text{Home Economics} = \frac{24}{80} \rightarrow \frac{24}{80} \times 360^\circ = 108^\circ$$

$$\text{Music} = \frac{10}{80} \rightarrow \frac{10}{80} \times 360^\circ = 45^\circ$$

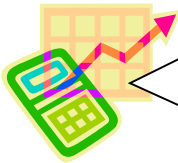
$$\text{Physics} = \frac{12}{80} \rightarrow \frac{12}{80} \times 360^\circ = 54^\circ$$

$$\text{PE} = \frac{6}{80} \rightarrow \frac{6}{80} \times 360^\circ = 27^\circ$$

Check that the total = 360°



Information Handling : Averages



To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value - the mean, the median and the mode.

Mean

The mean is found by adding all the data together and dividing by the number of values.

Median

The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode

The mode is the value that occurs most often.

Range

The range of a set of data is a measure of spread.

Range = Highest value - Lowest value

Example Class 1A scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

6, 9, 7, 5, 6, 6, 10, 9, 8, 4, 8, 5, 7

$$\begin{aligned} \text{Mean} &= \frac{6+9+7+5+6+6+10+9+8+4+8+5+7}{13} \\ &= \frac{90}{13} = 6.923... \quad \text{Mean} = 6.9 \text{ to 1 decimal place} \end{aligned}$$

Ordered values: 4, 5, 5, 6, 6, 6, 7, 7, 8, 8, 9, 9, 10
Median = 7

6 is the most frequent mark, so Mode = 6

$$\text{Range} = 10 - 4 = 6$$

Length

Length is how far it is from one end of something to the other or the distance between two points.

Language

metre	half-metre	centimetre
	half-centimetre	kilometre

Units of Length

10 millimetres (mm) = 1 centimetre (CM)

100 centimetre (cm) = 1 metre (m)

1000 metres (m) = 1 kilometre (km)

Estimate

Is your school tie shorter than one metre, longer than one metre or about the same length as one metre?

Is a door shorter than, longer than or about two and a half metres high?

Which is longer - two thousand metres or one and a half kilometres?

Can you draw a line $8\frac{1}{2}$ cm long.

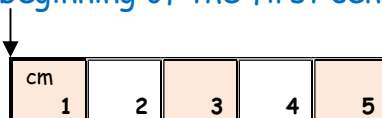
How long is your pencil?

Which is shorter: seven and a half kilometres or six thousand metres?



HINT

When you are measuring the length of something look at your ruler or tape measure carefully. Make sure you start measuring from the beginning of the first centimetre.



Weight



We use balances or scales to find out how heavy something is. We use bathroom scales to weigh ourselves. In the post office they use scales to weigh letters and parcels.

Language

kilogram half-kilogram gram
weighs about / less than / more than

Units of Weight

1000 grams (g) = 1 kilogram (kg)
1000 kg = 1tonne (metric)

Common questions

Example 1

Converting grams to kilograms

$$5264 \text{ g} = 5 \text{ kg } 264 \text{ g} = 5.264\text{kg}$$

$$3600\text{g} = 3\text{kg } 600\text{g} = 3.6\text{kg}$$

Example 2

Convert kilograms to grams

$$9\text{kg } 42 \text{ g} = 9042\text{g}$$

$$14.5\text{kg} = 14500\text{g}$$

$$9\text{kg} = 9000\text{g}$$

Example 3

Addition of mixed examples

$$780\text{g} + 4 \text{ kg } 234\text{g} + 9.5\text{kg} \quad (\text{Convert to g})$$

$$780\text{g} + 4234\text{g} + 9500\text{g} = 14 \text{ } 514\text{g}$$

$$14 \text{ } 154\text{g} = 14\text{kg } 514\text{g} \text{ or } 14.514\text{kg} \text{ (convert g to kg \& g or kg)}$$

Volume

The volume is the amount of space taken up by a 3D shape and this is sometimes called capacity.

Solid Volumes are measured in cubic centimetres and cubic metres (cm^3 and m^3)

Liquid volumes are measured in millilitres and litres. (ml and l)

Units of capacity (liquid)

1 litre (l) = 1000 millilitres (ml)

$\frac{1}{2}$ litre (l) = 500 millilitres (ml)

Units of capacity (solid)

1 m^3 = 1000 cm^3

Common questions

Example 1

Change millilitres to litres

$$3 \text{ l} = 3000 \text{ ml}$$

$$6.2 \text{ l} = 6200 \text{ ml}$$

$$8500 \text{ ml} = 8.5 \text{ l}$$

$$6254 \text{ ml} = 6.254 \text{ l}$$

Example 2

Write down the volume of liquid in the measuring tube?

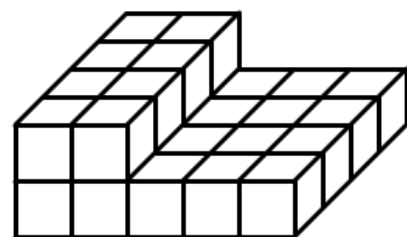
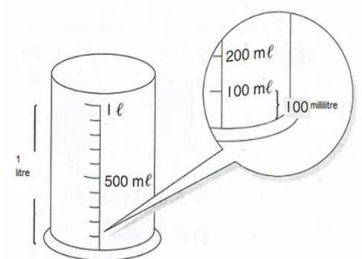
It is important to work out the scale, whether it is going up in 1ml, 2ml, 5ml, 10ml etc.

Example 3

Write down the volume of the shape in cm^3

Count all of the cubes, not forgetting the cubes under the first two rows.

Answer = 28 cm^3

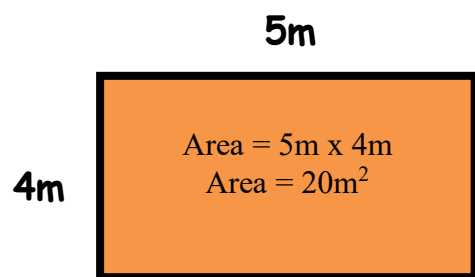


Area

The area of flat shape is defined as the amount of space it occupies and is generally measured in square centimetres (cm^2), square metres (m^2) and square kilometres (km^2)

The area of a rectangle can be measured by multiplying the length \times breadth

Area = Length \times Breadth

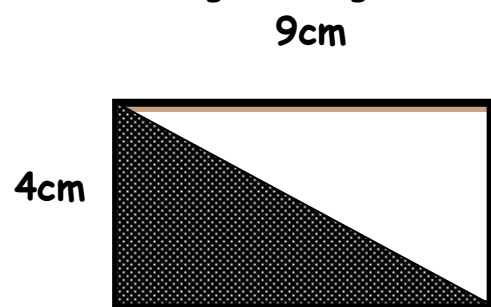


The area of right angled triangle can be found using the following two steps:-

First calculate the area of the surrounding rectangle

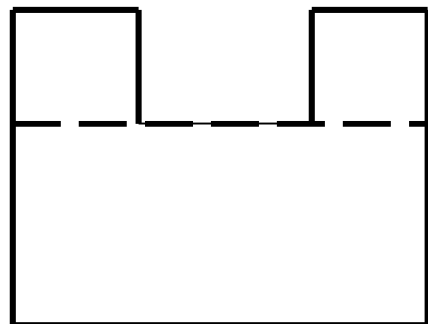
$$\text{Area} = 9 \times 4 = 36\text{cm}^2$$

Secondly, half this to find the area of the right angled triangle.



$$\text{Area} = \frac{1}{2} \text{ of } 36\text{cm}^2 = 18\text{cm}^2$$

The area of more complex shapes can be calculated by separating the shape into regular rectangles.




Mathematical literacy (Key words):

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: $12+76 = 88$
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Continuous Data	Has an infinite number of possible values within a selected range e.g. temperature, height, length
Data	A collection of information (may include facts, numbers or measurements).
Discrete	Can only have a finite or limited number of possible values. Shoe sizes are an example of discrete data because sizes 6 and 7 mean something, but size 6.3 for example does not.
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 $50 - 36 = 14$
Division (\div)	Sharing a number into equal parts. $24 \div 6 = 4$
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.

Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. Example: 10 is greater than 6. $10 > 6$
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. $15 < 21$.
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers
Median	Another type of average - the middle number of an ordered set of data
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average - the most frequent number or category
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example $6 \times 4 = 24$
Negative Number	A number less than zero. Shown by a minus sign. Example -5 is a negative number.
Numerator	The top number in a fraction.
Non Numerical data	Data which is non numerical e.g. favourite football team, favourite sweet etc.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1, 3, 5, 7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BODMAS
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).

Prime Number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60

60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

Multiplication Square

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100